Performance of Conical Jet Nozzles in Terms of Discharge Coefficient

K. Sheshagiri Hebbar,* K. Sridhara* and P. A. Paranjpe**

Summary

An attempt is made to obtain a simple, explicit and analytical expression for the discharge coefficient of conical convergent nozzles operating under varying pressure ratios. The theoretical results based on this approach have been compared with discharge coefficient of conical jet nozzles determined experimentally covering a range of pressure ratio up to 3.25. The theory predicts the correct trend of the variation of discharge coefficient with respect to nozzle pressure ratio and nozzle convergence angle. Satisfactory quantitative agreement with the experimental results is possible by applying a suitable correction factor for the boundary layer growth which is dependent on Reynolds number.

The effect of variation of semi-convergence angle, the effect of lip-rounding and the effect of stepped convergence on the discharge coefficient have also been investigated experimentally.

Nomenclature

- A Area of nozzle
- C_d Discharge coefficient of nozzle
- Cv Velocity coefficient
- CA Area contraction coefficient
- k Isentropic exponent
- m Mass flow rate
- M Mach number
- n Reciprocal of the index of power law for the velocity distribution in turbulent flow
- P Stagnation pressure
- p Static pressure
- Re Reynolds number of nozzle flow (based on exit diameter of nozzle)
- V Velocity
- « Convergence semiangle of nozzle
- P Inclination of streamlines near the lip w.r.t, nozzle axis, immediately after expansion at the nozzle exit plane
- P Density

Subscripts

- a Refers to actual values
- b Refers to ambient values
- c Refers to critical values
- e Refers to values at nozzle exit plane
- i Refers to ideal values (based on one-dimensional isentropic flow considerations)
- o Refers to values before the entry to nozzle.

I. Introduction

Conical convergent nozzles have been widely used in subsonic jet engines as a means to convert pressure energy into kinetic energy because of their inherent simplicity in construction. A knowledge of the discharge coefficient and its variation with operating pressure ratio and convergence semiangle is very important in the performance estimation of jet engines. Though there have been a number of attempts in the past to predict theoretically the variation of discharge coefficient, no simple, explicit and satisfactory expression has been obtained. However, experiments on the performance of conical jet nozzles in terms of flow and velocity coefficients were carried out systematically by Grey and Wilsted [1] as early as 1948.

In this paper a simple explicit and analytical expression for the discharge coefficient has been obtained for conical convergent nozzles operating under subcritical critical or supercritical condition. It is derived as function of overall operating pressure ratio and convergence semiangle of the nozzle. The follow assumptions have been made in the analysis ; (i) behind the nozzle exit plane, the magnitude of the velo vector is constant across the plane (and is given by o

*Scientist, Propulsion Division, National Aeronautical Laboratory, Bangalore.
**Head, Propulsion Division, National Aeronautical Laboratory, Bangalore,

Received on 21st Juno 1969,

dimensional isentropic treatment of the nozzle flow) but its direction varies across the plane, being parallel to the axis of the nozzle at the centre of the plane. (ii) The flow through the nozzle is turbulent. (iii) The nozzles have moderate convergence angles and short lengths so that the combined effect of favourable pressure gradient and nozzle pressure ratio on the boundary layer development within the nozzles may be accounted for by a suitable choice of the value of the reciprocal of the index of power law for the velocity distribution as discussed later (See Eq. 14).

The theoretical prediction has been compared with (actual discharge coefficient of conical jet nozzles determined experimentally in an investigation covering a range of pressure ratio upto 3.25. In addition to the effect of variation of convergence angle, the effect of (i) lip rounding (ii) addition of a short cylindrical piece and (iii) stepped convergence on the discharge coefficient have also been investigated.

2. Analysis

The nozzle discharge coefficient Cd is defined as the ratio of the actual mass flow rate through the nozzle to the ideal mass flow rate based on one-dimensional isentropic flow consideration. Thus

$$C_d = \frac{\text{Actual mass flow rate}}{\text{Ideal mass flow rate}}$$

i.e., $C_d = (\rho_{ea}/\rho_{el})\,(V_{ea}/V_{el})\,A_{ea}/A_{el})$

$$\text{or} \quad C_d = C_\rho \; C_V \; C_A \tag{1}$$

where C_ρ, C_V and CA may be referred to as the density coefficient, the velocity coefficient and the area contraction coefficient respectively.

The density coefficient will not be appreciably different from unity except in very high temperature jets in which chemical changes such as dissociation might occur resulting in an increase in the number of particles present. In the case of normal laboratory experiments it will be unity and we may write

$$C_d = C_V \, C_A \tag{2}$$

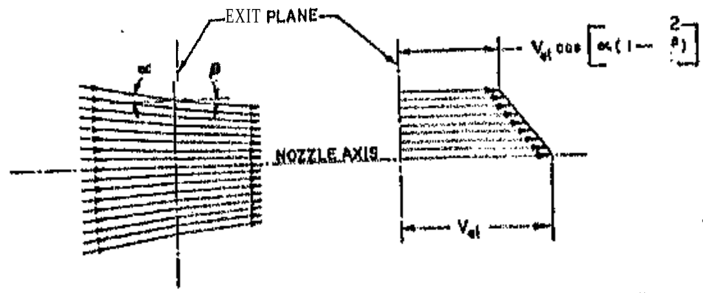
2.1. Velocity coefficient C_V :

Subcritical and supercritical cases will be considered separately :

2.1.1. Subcritical nozzle flow

$[0 \leq M_e \leq 1,\ p_e \simeq p_b]$:

Just upstream of the exit plane, the stream lines at he wall are inclined at an angle « to the nozzle axis, Immediately downstream of the exit plane the inclinatio of these stream lines, represented by p, depends on the no zle exit pressure and therefore on the nozzle pressure r io (Fig. 1a).



(a) VARIATION IN JET INCLINATION (b) VELOCITY VECTOR PARALLEL TO NOZZLE AXI:

Fig. 1a Subcritical Nozzle Flow, Subsonic Jet

At low pressure ratios ($M_e << 1$) the flow is incompressible and therefore just downstream of the exit pl ne streamlines at the wall may be assumed to come out a an angle $\beta = \alpha$, the inclination of streamlines decreasing as the axis of the nozzle is approached and finally vanisl ng on the axis. When the nozzle is just choked ($M_e = 1$), the nozzle exit pressure is exactly equal to the amb ent pressure and hence the streamlines become parallel to the axis immediately after the nozzle exit pl ne, i.e., $\beta = 0$.

At an intermediate pressure ratio ($M_e < 1$) the inclination of the streamlines at the wall immediately after he exit plane is assumed to be represented by a quadratic function of Mach number as observed experimentally indirectly :

$$\beta = \alpha\left(1 - M_e^2\right) \tag{3}$$

It is easy to verify that the Equation (3) satisfies he aforementioned boundary conditions on (4 at both he extremes $M_e = 0$ and $M_e = 1$.

Just at the exit plane, the mean inclination of he streamlines at the wall is given by $\left(\frac{\alpha+\beta}{2}\right) = \alpha\left(1 - \frac{M_e^2}{2}\right)$ and that of those at the axis zero. Hence, assuming a linear variation, the average axial component of all he streamlines for purposes of mass flow calculations is

$$V_{ea} = \left[V_{el} + V_{el} \cos\left\{ \alpha\left(1 - M_e^2/2\right)\right\}\right]/2$$

$$\text{or} \quad C_V = \frac{V_{ea}}{V_{ei}} \simeq \frac{1+\cos\left\{\alpha\left(1 - M_e^2/2\right)\right\}}{2} \tag{4}$$

Using the pressure Mach number relation

$$P_o/p_b = \left[1 + \frac{k-1}{2} M_e^2\right]^{\left(\frac{k}{k-1}\right)} \tag{5}$$

we get

$$C_V = \frac{1}{2}\left[1+\cos\left\{\alpha\left(1 - \frac{\left[P_o/p_b\right]^{\frac{k-1}{k}} - 1}{k-1}\right)\right\}\right], \quad 1 \leq P_o/p_b \leq \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \tag{6}$$

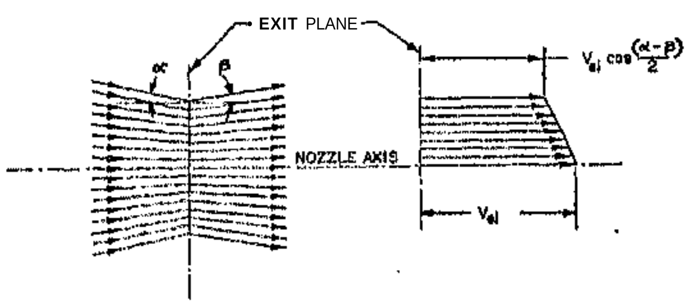
2.1.2. Supercritical nozzle flow $\left[M_e = 1,\ p_e > p_b,\ \left(\frac{P_o}{p_b}\right) > \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}\right]$:

In this case the jet expands just after leaving the nozzle according to Prandtl-Meyer expansion which is locally two-dimensional and is an isentropic process. Just after the exit plane, the streamlines near the wall deflect through an angle β given by (Fig. 1b)

$$\beta = \left(\frac{k+1}{k-1}\right)^{1/2} \tan^{-1}\left[\frac{(k-1)\left(M_b^2 - 1\right)}{k+1}\right]^{1/2} - \tan^{-1}\left(M_b^2 - 1\right)^{1/2} \tag{7}$$

where Mb is the Mach number of the free jet reached after its expansion to ambient pressure p_b and is given by

$$M_b = \left\{\left(\frac{2}{k-1}\right)\left[(P_o/p_b)^{\frac{k-1}{k}} - 1\right]\right\}^{1/2} \tag{8}$$



(a) VARIATION IN JET INCLINATION (b) VELOCITY VECTOR PARALLEL TO NOZZLE AXIS

Fig. 1b Supercritical Nozzle Flow

Therefore, just at the exit plane, the mean inclination of the streamlines at the wall is $\left(\frac{\alpha-\beta}{2}\right)$, for $\beta \leq \alpha$,

$$\text{Hence} \quad C_V = \frac{V_{ea}}{V_{ei}} = \frac{1}{2}\left[1 + \cos\left(\frac{\alpha-\beta}{2}\right)\right] \tag{9}$$

$$\text{or } C_V = \frac{1}{2}\left[1 + \cos\left\{\frac{\alpha - \left(\frac{k+1}{k-1}\right)^{1/2}\tan^{-1}\left[\frac{2}{k+1}\left\{\left(\frac{P_o}{p_b}\right)^{\frac{k-1}{k}} - 1\right\} - \frac{k-1}{k+1}\right]^{1/2} + \tan^{-1}\left[\frac{2}{k-1}\left\{\left(\frac{P_o}{p_b}\right)^{\frac{k-1}{k}} - 1\right\} - 1\right]^{1/2}}{2}\right\}\right] \tag{10}$$

Note that Equation (10) is valid for $(P_o/p_b) \geq \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} = 1.892$ for $k = 1.4$ and $\beta \leq \alpha$.

2.2. Area contraction Coefficient C_A :

The flow through the nozzle is assumed to be turbulent and to be represented at the nozzle exit by a power law of the form

$$\frac{v}{V} = \left(\frac{y}{R}\right)^{\frac{1}{n}} \qquad (11)$$

where R is the radius of the nozzle at the exit plane (Fig. 2a). If δ^* is the boundary-layer displacement thickness at the exit plane, then from continuity considerations, it follows that

$$\int_{y=0}^{R} (V-v)2\pi(R-y)dy = \pi[R^2-(R-\delta^*)^2]\, V \qquad (12)$$

From Equations (12) and (11), neglecting small quantities, we get

$$\left(\frac{\delta^*}{R}\right) = \frac{(3n+1)}{2\,(n+1)\,(2n+1)} \qquad (13)$$

Note that the corresponding relation for the two-dimensional plane flow is

$$\left(\frac{\delta^*}{R}\right) = \frac{1}{(n+1)}$$

The effective radius of the nozzle at the exit plane (Fig. 2b) is reduced to $(R-\delta^*/\cos\alpha)$ and therefore the area contraction coefficient may be written

$$C_A = \frac{A_{ea}}{A_{el}} = \frac{\pi\,(R-\delta^*/\cos\alpha)^2}{\pi R^2} = (1-\delta^*/R\cos\alpha)^2$$

$$\text{i.e.,}\quad C_A = \left[1 - \frac{(3n+1)}{2\,(n+1)\,(2n+1)\cos\alpha}\right]^2 \qquad (14)$$

It can be seen that Cv is a strong function of n but a weak function of a. The choice of the index n depends on the type of flow, i.e., accelerating or decelerating, and the Reynolds number of flow [2], It increases with Re and, as shown in Fig. 20,3, page 505 of Ref. [2], has a value of 10 for turbulent flow through smooth pipes at a Reynolds number (based upon pipe diameter) of 10^6. Accelerating flows such as those present in convergent nozzles have retarding effect on boundary layer growth inside the nozzle. In such cases n should assume extremely high values (see velocity distribution plotted in Fig. 22.1, page 568 of Ref. [2]). One method of determining n would be to measure the velocity profile at the nozzle exit and to compare this with equation (11). In the absence of any such information, the variation of n with respect to pressure ratio is assumed according to the following table, whose values have been arrived at after a thorough consideration of the experimental results on discharge coefficient reported in section 4 and are recommended for use in case of convergent nozzles (i.e. in favourable pressure gradients) operating at a Reynolds number around 10^6.

2.3. Nozzle discharge coefficient C_d :

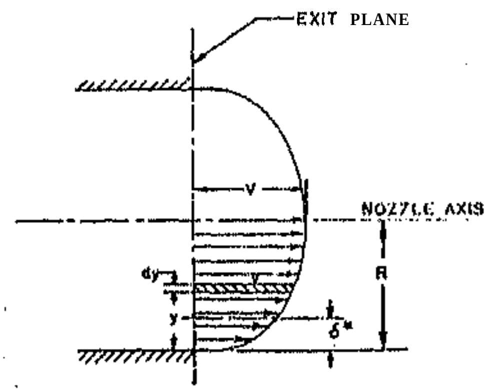
Using expressions for Cv CA derived earlier and Equation (2) we may write the following expressions for C_d :

For subcritical flow

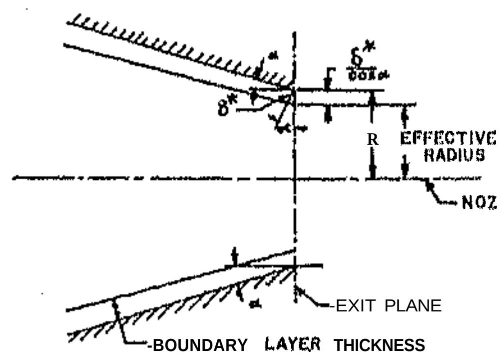
$$\left[1 < (P_0/p_b) < \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} = 1.892 \text{ for } k = 1{,}4\right]:$$

Table : Suggested values of n (for k = 1,4)

P_0/p_b	1.1.	1,3.	1,6	1.892	>1.892
n	7	10	15	20	30



(a) Velocity Distribution for Turbulent flow



(b) Effective Radius at the Nozzle Exit Plane

Fig, 2 Boundary layer development In the nozzle

$$C_d = \frac{1}{2}\left[1+\cos\left\{\alpha\left(\frac{2}{k-1}\left[\left(\frac{p_0}{p_b}\right)^{\frac{k-1}{k}}-1\right]\right)^{1/2}\right\}\right]\left[1-\frac{(3n+1)}{2(n+1)(2n+1)}\cos\alpha\right]^2 \qquad (15)$$

For supercritical **flow**

$$\left[\left(\frac{p_0}{p_b}\right) > \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} = 1.892 \text{ for } k = 1.4\right]:$$

$$C_d = \frac{1}{2}\left[1+\cos\left\{\frac{\alpha\left(\frac{k+1}{k-1}\right)^{1/2}\tan^{-1}\left[\frac{2}{k+1}\left\{\left(\frac{p_0}{p_b}\right)^{\frac{k-1}{k}}-1\right\}-\frac{k-1}{k+1}\right]^{1/2}+\tan^{-1}\left[\frac{2}{k-1}\left\{\left(\frac{p_0}{p_b}\right)^{\frac{k-1}{k}}-1\right\}-1\right]^{1/2}}{2}\right\}\right]$$
$$\times\left[1-\frac{(3n+1)}{2(n+1)(2n+1)}\cos\alpha\right] \qquad (16)$$

Either of equations (15) or (16) is valid for critical flow. The limitations on Equation (10) referred to earlier are equally applicable to Equation (16).

3. Experimental set-up

Some tests for the determination of the discharge coefficient of conical nozzles were carried out at the N.A.L. The experimental set-up is shown schematically in Fig, 3. The mass flow rate of air, supplied at approximately room temperature, was determined by the use of a standard B.S.S. orifice plate with D and $\frac{D}{2}$ tappings [3]. The orifice upstream pressure was measured to an accuracy of 0.1 psi and the pressure drop to an accuracy of a tenth of an inch of mercury.

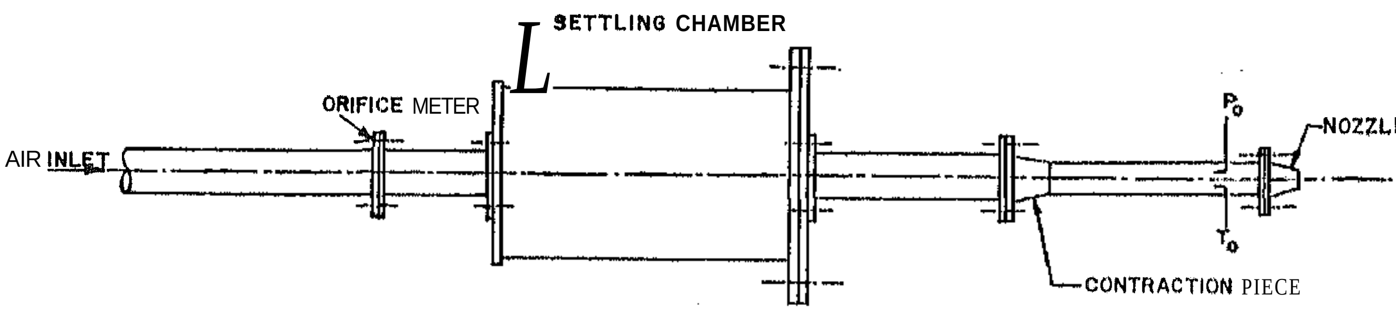


Fig. 3 Schematic diagram of the experimental set-up

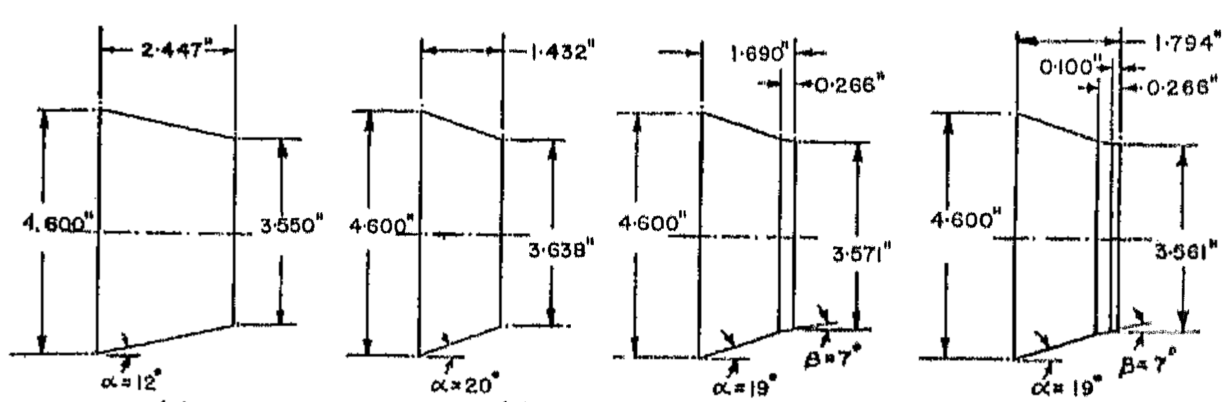


Fig. 4 Details of nozzle configurations

Isentropic mass flow rate was calculated on the basis of the measured stagnation condition upstream of nozzle and ambient conditions for the given geometrical dimensions of the nozzle. The discharge coefficient was determined as the ratio of actual mass flow rate measured to the above computed value of isentropic mass flow rate.

All nozzles were machined from cast aluminium blocks. Fig. 4 gives the nozzle configurations and dimensions*. All the nozzles have an inlet diameter of 4.60" and a nominal exit diameter of 3.60" (corresponding to an area ratio of 0.6).

4. Results aud Discussion

Fig. 5 depicts the variation in discharge coefficient with pressure ratio for various convergence semiangles. The experimental results correspond to a flow Reynolds number (based on exit plane diameter of the nozzle) around 10^6. Several features of the behaviour of the discharge coefficient w.r.t. pressure ratio are noteworthy in Fig 5. First, for a given convergence semiangle, the discharge coefficient increases with the overall pressure ratio, Secondly, it increases with decreasing convergence semiangle. Thirdly, it attains high values in the supercritical range. It can be observed that the theoretical curves follow very closely the experimental results. Experimental results were found to be consistently reproducible and the scatter was always less than ± 1%. In general, the experimental values of Ca in these tests are lower by 2 to 3% than those presented in Ref. [1]. It may be noted that the present theory describes the behaviour of the discharge coefficient w.r.t, pressure ratio very satisfactorily.

The extremely high values of n (specially for $P_0/p_b \geqslant 1.892$) suggest the existence of only thin turbulent boundary layers in the experiments. It is expected that these values would be realistic in situations where there would be only a rather thin turbulent boundary layer at the exit, as is likely to be the case in nozzles of jet engines.

The fact that the nozzle discharge coefficient increases with the decrease of convergence semiangle is of special significance because it indicates that the performance of covergent nozzles with small convergence angles is superior to those with large convergence angles. This means that it should be possible to improve the performance of nozzles by rounding of the nozzle exit or by adding a short length of straight pipe to the nozzle exit, taking care to allow for a smooth transition at the junction. Other important inference is that even when

*Configurations which were of interest to the Gas Turbine Research Establishment, Bangalore, were selected for experimental investigations.

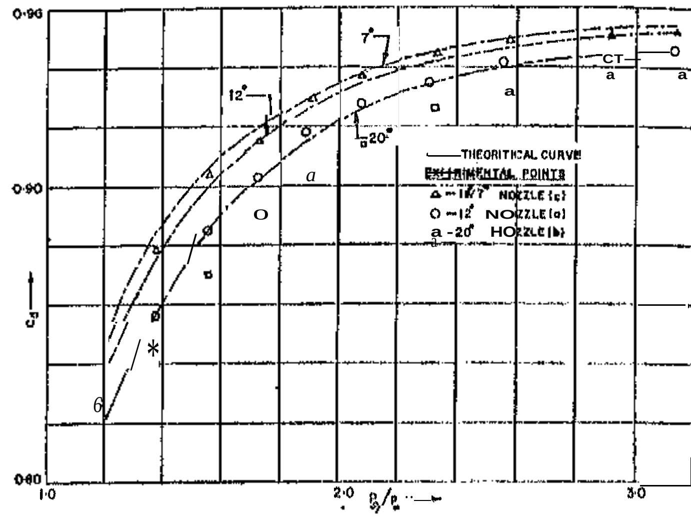


Fig. 6 Variation of discharge coefficient with pressure ratio

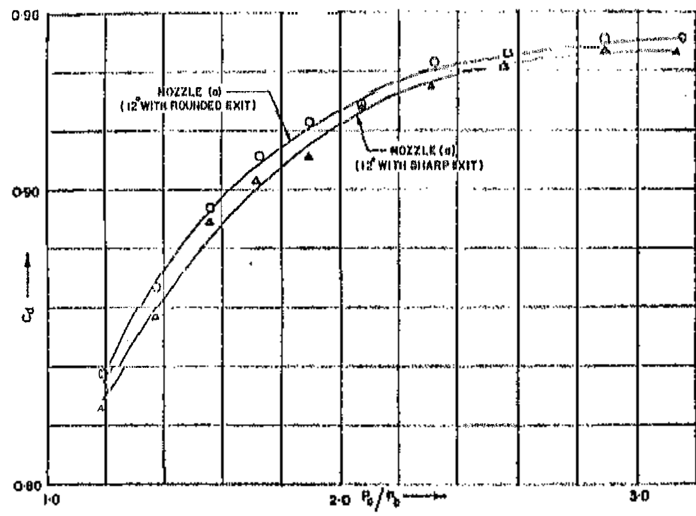


Fig. 6 Effect of lip-rounding at the nozzle exit

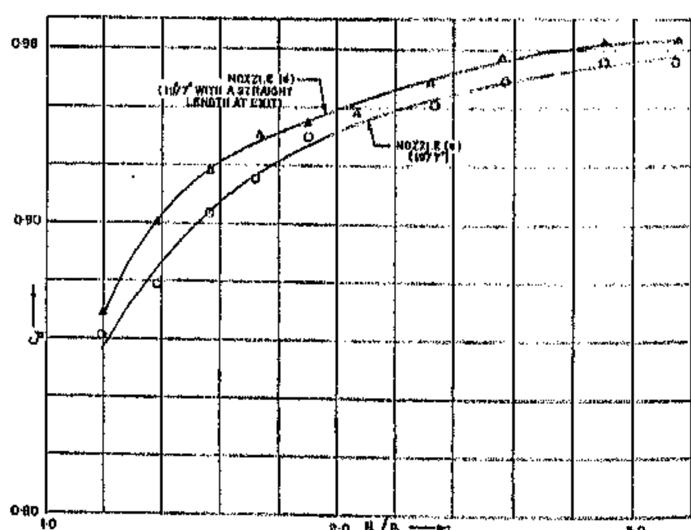


Fig. 7 Effect of adding a short length of straight pipe to the nozzle exit

the length of the nozzle is limited for one reason or the other it should be possible to improve the performance of the nozzle by having a larger convergence initially, decreasing the same in one or several steps as the case may be. It is very important to see that there is a smooth transition at the junction of such "steps" because, otherwise, the flow may tend to separate. If this is ensured, the final convergence angle will be the deciding factor in determining the performance of such stepped nozzles.

These theoretical predictions regarding the effect of (i) rounding of the nozzle exit, (ii) adding a short length of straight pipe to the nozzle exit and (iii) using stepped nozzles are very well borne out by experimental observations shown plotted in Fig. 6, 7 and 5 respectively.

5. Conclusions

A simple explicit analytical expression has been developed for the discharge coefficient of conical convergent nozzles operating under various pressure ratios, The agreement between the theory and experiment is quite good. The following conclusions may be drawn :

(i) The nozzle discharge coefficient increases with increasing overall operating pressure ratio but decreases with increasing convergence angle.

(ii) The discharge coefficient is fairly high in the supercritical range.

(iii) The performance of a convergent nozzle can be greatly improved by (a) rounding of the nozzle exit (b) adding a short cylindrical piece to the nozzle exit or (c) stepping the convergence. The stepped nozzle with a larger initial convergence angle and a smaller final convergence angle may be employed without loss of performance in situations where the length of the nozzle may be limited, e.g. turbo jets.

***6.* Acknowledgement**

The authors are indebted to Wg. Cdr. Radhakrishnan of G.T.R.E. Bangalore, for his active participation and help in carrying out the experiments.

7. References

1. GREY, R. E. and WILSTED, H.D., "Performance of conical jet nozzles in terms of flow and velocity coefficients," NACA TN 1757 (1948).

2. SCHLICHTING, H , "Boundary layer theory," Mc.Graw Hill Book Company Inc., New York, 4th Edition, pp. 505 & 568.

3. "Flow measurement," British Standard Code, B.S. 1042, (1943),

—: o :—